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# Value of Information-based Inspection, Monitoring, and Damage Detection System Planning

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**ABSTRACT:** Inspections, Structural Health Monitoring (SHM), and Damage Detection System (DDS) are integral parts of structural integrity management (SIM) to ensure the structural performance and the required safety. This paper describes novel approaches on how the outcomes of Structural Health Monitoring (SHM) can be modeled, predicted, and combined with inspection planning in the frame of pre-posterior decision analysis and utilized for an optimization of the SIM. The SIM is hereby formulated for a deteriorating structural system subjected to extreme loading with the objective to minimize the risk and expected costs for an envisaged and extended service life. Due to the complexity of the decision analysis for this type of problem, several simplifications and a decision rule are derived and discussed to reduce the computational costs. An application example with a deteriorating brittle Daniels system is presented and discussed. The example shows SHM and DDS can be utilized together with inspections to further optimize SIM strategy and reduce the expected costs over the service life.

## 1. INTRODUCTION

As part of structural integrity management (SIM), inspection and monitoring provide valuable information with respect to the structural system performance. Approaches to optimize inspection planning by utilizing the Bayesian decision analysis framework, especially the Value of Information (VoI) theory, had been studied in the last decade (see e.g., Straub (2004)). Similar studies had been conducted to optimize the utilization of SHM (see e.g., Thöns (2017)) and DDS (see e.g., Thöns et al. (2018)).

This paper addresses the quantification of the value of inspections when combined with SHM and DDS in order to optimize SIM strategy. In Section 3, the modeling approaches for inspection, SHM and DDS are presented. A novel approach for SHM modeling by utilizing the stress range model uncertainty based on the approach by Agusta and Thöns (2018) is introduced. Three SIM strategies are in-

vestigated in this study: (1) only inspections, (2) inspections with SHM, and (3) inspections with DDS (see Section 4). Deteriorating brittle Daniels system with four components subjected to an external load is utilized in the case study (see Section 5). The results show a high reduction in total expected costs by utilizing the SIM strategies compared to without SIM. It is also observed that the outcome of SHM/DDS can significantly influence future inspection planning by changing the inspection frequency and inspection interval.

## 2. STRUCTURAL RELIABILITY MODELING

### 2.1. Probability of Fatigue and System Failure for Brittle Daniels System

Structural systems performance may deteriorate due to the accumulation of damage during the service life. With increasing damage, the component resistance is decreasing due to the loss of cross-sectional area and may cause component failure.

The component failure's limit state function is

$$g_{F,i}(\mathbf{X}, t) = M_{R,i}R_i(t) - M_{L,i}L_i \quad (1)$$

$L_i$  is the maximum component load, and  $M_{R,i}$  and  $M_{L,i}$  are the resistance and the load model uncertainty, respectively.  $\mathbf{X}$  is a vector of random variables that influence the deterioration. The component resistance  $R_i$  deterioration is modeled as follows

$$R_{i,i}(t) = R_{0,i} \left( 1 - d_{R,i} \frac{\delta_i(t)}{\delta_{c,i}} \right) \quad (2)$$

where  $R_0$  is the initial resistance,  $\delta_i(t)$  is the crack size at time  $t$ , and  $\delta_{c,i}$  is the critical crack size.  $d_R$  is the damage scaling function:

$$d_{R,i} = \begin{cases} 1 & \delta_i(t)/\delta_{c,i} \leq 1 \\ \frac{\delta_{c,i}}{\delta_i(t)} & \delta_i(t)/\delta_{c,i} > 1 \end{cases} \quad (3)$$

The damage event  $D$  is defined with the following limit state function:

$$g_{D,i}(\mathbf{X}, t) = 1 - \frac{\delta_i(t)}{\delta_{c,i}} \quad (4)$$

Real structures often consist of redundant components and the Daniels system (see Figure 1) may be used to idealize such structural systems. Assuming brittle material behavior, the system failure limit state function for brittle Daniels system is written as follows:

$$g_{FS}(\mathbf{X}_S, t) = \bigcap_{i=1}^{N_C} \left\{ (N_C - i + 1) \hat{M}_{R,i} \hat{R}_i(t) - M_{L_S} L_S \right\} \quad (5)$$

where  $N_C$  is the number of components in the Daniels system, and  $\hat{M}_{R,i}$  and  $\hat{R}_i(t)$  are the realizations of  $M_{R,i}$  and  $R_i(t)$ , respectively. Note that  $\hat{M}_{R,i} \hat{R}_i(t)$  need to be ordered in ascending order to calculate Eq. 5, e.g.,  $\hat{M}_{R,1} \hat{R}_1(t) \leq \dots \leq \hat{M}_{R,N_C} \hat{R}_{N_C}(t)$ . The probability of system failure  $P(F_S(t))$  is then calculated as follows

$$P(F_S(t)) = \int_{g_{FS}(t) \leq 0} f_{\mathbf{X}_S}(\mathbf{x}_S) d\mathbf{x}_S \quad (6)$$

where  $f_{\mathbf{X}_S}$  is the joint distribution of random variable  $\mathbf{X}_S$ .

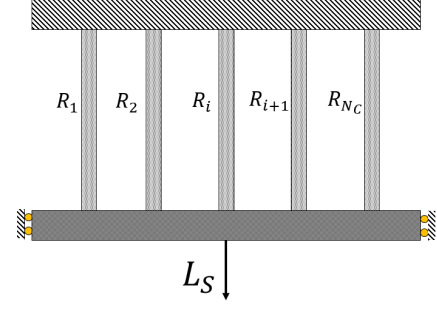


Figure 1: An illustration of Daniels system with  $N_C$  components subjected to an external system load  $L_S$ .

## 2.2. Fatigue crack growth model

The damage is assumed caused by the fatigue crack growth and modeled with Paris' law. The crack size at time  $t$  at an infinite panel is calculated as follows (Ditlevsen and Madsen, 1996)

$$\delta(t) = \left( \delta_0^{1-\frac{m}{2}} + \left( 1 - \frac{m}{2} \right) C \left( B_{SIF} B_{\Delta S} \Delta S_e \sqrt{\pi} \right)^m \nu t \right)^{\frac{1}{1-\frac{m}{2}}} \quad (7)$$

where  $C$  and  $m$  are the empirical model parameters,  $\delta_0$  is the initial crack size,  $\nu$  is the cycle rate, and  $B_{SIF}$  and  $B_{\Delta S}$  are the model uncertainties of the stress intensity factor and the stress range, respectively. The equivalent stress range  $\Delta S_e$  is defined as follows:

$$\Delta S_e = k \left[ \Gamma \left( 1 + \frac{m}{\lambda} \right) \right]^{1/m} \quad (8)$$

where  $k$  and  $\lambda$  are the Weibull scale and shape parameter and  $\Gamma(\cdot)$  is the complete gamma function. Therefore, the random space is defined as  $\mathbf{X} = [\delta_0, m, C, \Delta S, B_{SIF}, B_{\Delta S}, k, \lambda]$

## 3. STRUCTURAL INFORMATION MODELING

### 3.1. Inspection and Damage Detection System Modeling

There are two outcomes of inspection and DDS: no indication ( $I_1$ ) or indication ( $I_2$ ) of damage. Inspection by non-destructive test (NDT) can be modeled by utilizing the signal distribution,  $f_S(s)$  (Gandossi and Annis, 2010). With this method, the probability of indication given damage  $\delta$  can be calculated as follows

$$P(I_2|\delta) = \int_{th_s}^{\infty} f_S(s) ds \quad (9)$$

where  $th_S$  is the signal threshold. The signal threshold may be defined by using the noise distribution  $f_{S_R}(s_R)$  at undamaged component and taking into account the probability of false indication (PFI) as seen in Eq. 10.

$$th_S : \int_{th_S}^{\infty} f_{S_R}(s_R) dn = PFI \quad (10)$$

where  $PFI$  is assumed to be known. The threshold can also be treated as optimization parameter to maximize the value of information and action (VoIA).

The marginal probability of no indication ( $I_1$ ) is calculated as follows

$$P(I_1) = \int_{g_{I_1}(\mathbf{X}, t) \leq 0} P(I_1|\delta) f_{\delta}(\delta) d\delta \quad (11)$$

where  $g_{I_1}(\mathbf{X}, t)$  is the no indication limit state function (Hong, 1997):

$$g_{I_1}(\mathbf{X}) = P(I_2|\delta) - z \quad (12)$$

where  $z$  is a uniformly distributed random variable.

DDS can be modeled with a multivariate probability of indication based on the damage indicator value (DIV) distribution for each damage states (Thöns et al., 2018). The multivariate probability of indication contains the uncertainties of the measurement precision, DDS operation, model uncertainty of the DIV derivation, and algorithm precision. The probability of no damage indication given damage vector  $\delta$  is given as follows

$$P(I_2^{DDS}|\delta) = \int_{th_V}^{\infty} f_V(v|\delta) dv \quad (13)$$

where  $\delta = [\delta_i, \dots, \delta_{N_C}]$  and  $th_V$  is the DIV threshold and may be derived from the DIV distribution in undamaged system by taking into account the probability of false indication  $PFI_S$

$$th_V : \int_{th_V}^{\infty} f_{V_R}(v_R|\delta = 0) dv_R = PFI_S \quad (14)$$

Similar to inspection approach, the threshold can also be treated as optimization parameters to maximize VoIA. The probability of no damage indication is defined as follows

$$P(I_1^{DDS}) = \int_{g_{I_1^{DDS}}(\mathbf{X}_S) \leq 0} P(I_1^{DDS}|\delta) f_{\delta}(\delta) d\delta \quad (15)$$

where  $g_{I_1^{DDS}}(\mathbf{X}_S)$  is calculated as follows

$$g_{I_1^{DDS}}(\mathbf{X}_S) = P(I_2^{DDS}|\delta) - u \quad (16)$$

$u$  is a uniformly distributed random variable.  $P(I_2^{DDS}|\delta)$  is calculated by utilizing the mixture of DIV distribution in undamaged/reference state and in damaged state as illustrated in Figure 2.

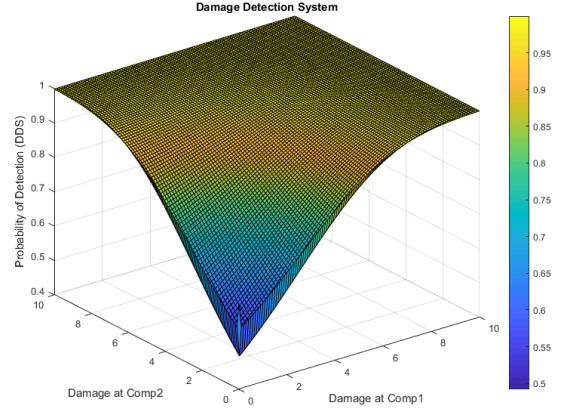


Figure 2: An example of POD distribution of DDS for sensors placed at two components.

### 3.2. Structural Health Monitoring Modeling

SHM can be installed to monitor specific structural properties (such as e.g., vibration). Operational modal analysis (OMA) can be used in conjunction with structural model to estimate the stress ranges. OMA is utilized to estimate the mode shape vectors from the monitoring data (e.g., vibration measurement) which then expanded to cover the entire model (Brincker and Ventura, 2015). Therefore, the calibrated model will give better estimates of structural responses (e.g., stress ranges). The stress ranges estimate obtained from OMA can be viewed as the possible realizations of the stress ranges model uncertainty. Before SHM implementation, the SHM outcome is unknown but with a known monitoring uncertainty  $U$  and may be predicted by utilizing the stress range model uncertainty  $B_{\Delta S}$  and two thresholds  $\eta_1$  and  $\eta_2$ . Each threshold is associated with a target damage probability  $P_D^T$  as shown in Eq. 17.

$$\begin{aligned} \eta_1 : P(D|\eta_1(t)) &= P_{D,1}^T \\ \eta_2 : P(D|\eta_2(t)) &= P_{D,2}^T \end{aligned} \quad (17)$$

where  $P_{D,1}^T < P_{D,2}^T$ . If the component damage probability is lower than  $P_{D,1}^T$ , the SHM will indicate that the component is better than expected ( $Z = Z_1$ ). If it is in-between  $P_{D,1}^T$  and  $P_{D,2}^T$ , the component is performing as designed ( $Z = Z_2$ ) and a bad performance ( $Z = Z_3$ ) if the damage probability exceeds  $P_{D,2}^T$ . Therefore, the outcomes can be defined as three indication events as follows (see e.g. Agusta and Thöns (2018)):

$$\begin{aligned} Z_1 : \hat{B}_{\Delta S} &\leq \eta_1 \\ Z_2 : \eta_1 &< \hat{B}_{\Delta S} < \eta_2 \\ Z_3 : \hat{B}_{\Delta S} &\geq \eta_2 \end{aligned} \quad (18)$$

As in inspection and DDS approach, the thresholds can also be treated as optimization parameters to maximize VoIA. The marginal probability of each of the outcomes is calculated from the stress range model uncertainty distribution as follows, see Figure 3:

$$P(Z_k) = \int_{\eta_{k-1}}^{\eta_k} f_{B_{\Delta S}}(B_{\Delta S}) dB_{\Delta S} \quad (19)$$

where  $\eta_0 = 0$ ,  $\eta_4 = \infty$ , and  $k = 1, 2, 3$ .

### 3.3. Structural System Updating

The outcome of an inspection can be used to update the probability of system failure by utilizing the Bayes' rule. Given no indication of damage during an inspection campaign, the updated system failure probability is given as follows:

$$P(F_S | I_1^C) = \frac{P(F_S \cap I_1^C)}{P(\bar{I}^C)} \quad (20)$$

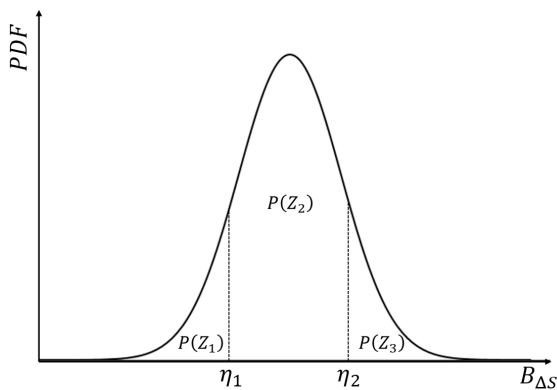


Figure 3: The probability density function of  $B_{\Delta S}$  with two thresholds.

where  $I_1^C$  is defined as

$$I_1^C = \bigcap_{i=1}^{N_C} I_{1,i} = \bigcap_{i=1}^{N_C} \{g_{I_{1,i}}(\mathbf{X}) \leq 0\} \quad (21)$$

If DDS is installed before the inspection campaign is performed, the system failure probability is updated according to Eq. 22.

$$P(F_S | I^{DDS}, I_1^C) = \frac{P(F_S \cap I^{DDS} \cap I_1^C)}{P(I^{DDS} \cap I_1^C)} \quad (22)$$

$I^{DDS}$  is the possible outcome of DDS, which can be no indication ( $I_1^{DDS}$ ) or an indication ( $I_2^{DDS}$ ) of system damage.

If SHM is installed at a component before the inspection campaign, the system failure probability is updated as follows:

$$P(F_S | Z, I_1^C) = \frac{P(F_S \cap Z \cap I_1^C)}{P(Z \cap I_1^C)} \quad (23)$$

where  $Z$  is the possible outcome of SHM (see Eq. 18).

## 4. VALUE OF INSPECTIONS, MONITORING, AND DAMAGE DETECTION SYSTEM

The Value of Information (VoI) theory has been used to quantify the value of inspection and monitoring (see e.g. Agusta et al. (2017), Thöns et al. (2018)). Detailed explanation of the pre-posterior decision analysis can be seen in Raiffa and Schlaifer (1961). In order to calculate the value of inspections before the actual inspections are performed, two scenarios are considered: the do-nothing scenario (prior) and the scenario with inspections (pre-posterior). The value of information and action is then calculated as the difference between the expected total costs in the prior scenario  $C_0$  and the pre-posterior scenario  $C_1$  as follows:

$$VoIA = C_0 - C_1 \quad (24)$$

In the prior scenario, no inspections and repair is performed on the structural system. Therefore, the prior expected total costs  $C_0$  is simply the sum of expected failure costs over the service life, see Eq. 25.

$$C_0 = \left( \sum_{i=1}^{N_C} E[C_{F,i}] \right) + E[C_{FS}] \quad (25)$$

$E[C_{F,i}]$  and  $E[C_{FS}]$  are the expected component and system failure costs over the service life, respectively.

In this paper, three strategies of SIM are considered: (1) only inspections, (2) inspections and monitoring, and (3) inspections and DDS (see Table 2). In the first strategy, inspections are performed one year before the annual system failure probability exceeds the threshold  $\Delta P^{th}(F_S)$ . After the inspections, repair can be conducted at the inspected components.

The expected total costs of the first SIM strategy is as follows:

$$C_{1,1} = \left( \sum_{i=1}^{N_C} E[C_{I,i}] + E[C_{R,i}] + E[C_{F,i}] \right) + E[C_{FS}] \quad (26)$$

$E[C_{I,i}]$  and  $E[C_{R,i}]$  are the expected inspection and repair costs, respectively.

The second strategy combines inspections with monitoring. The expected total costs for this strategy is calculated as follows:

$$C_{1,2} = \left( \sum_{i=1}^{N_C} E[C_{Insp,i}] + E[C_{F,i}] \right) + E[C_{SHM}] + E[C_{FS}] \quad (27)$$

$$E[C_{Insp,i}] = E[C_{I,i}] + E[C_{R,i}] \quad (28)$$

$E[C_{SHM}]$  is the expected SHM costs.

The third strategy is to utilize DDS together with inspections. The expected total costs is calculated as follows:

$$C_{1,3} = \left( \sum_{i=1}^{N_C} E[C_{Insp,i}] + E[C_{F,i}] \right) + E[C_{DDS}] + E[C_{FS}] \quad (29)$$

where  $E[C_{DDS}]$  is the expected DDS costs.

## 5. CASE STUDY

### 5.1. Structural System Description

A brittle Daniels system with  $N_C = 4$  hotspots and 20 years service life is considered. The annual maximum system load  $L_S$  is Weibull distributed and resisted by equally by 4 components with initial resistance  $R_{0,i}$ . The expected value of the initial resistance is calibrated to  $10^{-5}$  probability of component failure in undamaged state. The correlation coefficient of the component resistance and fatigue model parameters is assumed 0.6. The summary of the random variables is shown in Table 1.

Table 1: Summary of the random variables for system and FM modeling.

Var.	Dim.	Dist.	Exp.	Std.
$M_R$	-	LN	1	0.15
$R_0$	-	LN	Cal.	$0.15\mu_{R_0}$
$M_{L_S}$	-	LN	1	0.1
$L_S$	-	WBL	9.1	6.53
$T_{SL}$	year	-	20	-
$\delta_0$	mm	EXP	0.11	-
$\delta_c$	mm	-	8	-
$\ln C$	N and mm	N	-29.97	0.5095
$m$	-	-	3.0	-
$B_{SIF}$	-	LN	1.0	0.1
$B_{\Delta S}$	-	LN	1.0	0.2
$\ln k$	N and $mm^2$	N	2.1	0.22
$\lambda$	-	N	0.8	0.08
$\nu$	1/year	-	$10^7$	-

LN:Lognormal, WBL:Weibull, N:Normal, EXP:Exponential

### 5.2. SIM Strategy

The inspection is performed at all hotspots by utilizing the constant threshold approach. The threshold of the annual system failure probability  $\Delta P^{th}(F_S)$  is assumed  $3 \cdot 10^{-5}$  (unless varied). The repaired components are assumed to behave as undetected components. The signal distribution is Normal distributed with the expected value and the standard deviation are defined as follows:

$$\begin{aligned} \mu_{s,i}(t) &= 1.0 + 0.1 \cdot \delta(t) \\ \sigma_{s,i}(t) &= 0.15 - 0.1 \cdot \delta(t) \end{aligned} \quad (30)$$

The noise distribution is Normal distributed with the expected value of 1 and the standard deviation of 0.5. The signal threshold  $th_s$  is associated with probability of false indication of 0.01.

The stress range monitoring is performed at 1 component for one year at  $t = t_{I,1} - 1$  where  $t_{I,1}$  is the time of the first inspection. For simplicity, the thresholds  $\eta_1$  and  $\eta_2$  are associated with two target component damage probabilities of  $3 \cdot 10^{-4}$  and  $2 \cdot 10^{-3}$ , respectively. The measurement uncertainty

Table 2: The decision scenario and variables for the case study.

		Description
Structure	Type	Generic redundant structural system
	Life-cycle phase	Operation
	Performance	A brittle Daniels system with 4 components subjected to fatigue deterioration and an extreme system load
	Performance boundaries	The annual system failure probabilities should not exceed a given threshold.
Decision scenario	Decision maker	Operator/owner of the structures.
	Decision point in time	At the start of service life
	Objective	Minimization of expected costs over the service life
Decision Variables	Actions	Do-nothing      Repair
	Action parameters	The component is not repaired <ul style="list-style-type: none"> <li>➤ The repaired components behave as undetected components</li> <li>➤ Repair is only performed if damage is detected</li> </ul>
	Information acquirement/SIM strategies	<ul style="list-style-type: none"> <li>▪ Inspections at all components with constant threshold approach</li> <li>▪ Inspections with Structural Health Monitoring (SHM) system <ul style="list-style-type: none"> <li>• SHM is installed at one component for one year and at the same year as the first inspection</li> </ul> </li> <li>▪ Inspections and Damage Detection System (DDS) <ul style="list-style-type: none"> <li>• DDS is installed at all components for one year and at the same year as the first inspection</li> </ul> </li> </ul>

$U$  is assumed Normal distributed with the expected value of 1 and the standard deviation of 0.05.

Sensors with damage detection system are installed for one year at  $t = t_{I,1} - 1$ . The DIV distribution in undamaged state is assumed to follow multivariate Normal with the following expected value and covariance matrix:

$$\begin{bmatrix} \mu_{V_{R,1}} \\ \vdots \\ \mu_{V_{R,4}} \end{bmatrix} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}, \mathbf{\Sigma}_{V_R} = \begin{bmatrix} 0.5 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0.5 \end{bmatrix} \quad (31)$$

The DIV distribution in damaged state is multivariate Normal distributed with following parameters:

$$\begin{bmatrix} \mu_{V_1} \\ \vdots \\ \mu_{V_4} \end{bmatrix} = \begin{bmatrix} \mu_{s,1}^{DDS} \\ \vdots \\ \mu_{s,4}^{DDS} \end{bmatrix}, \mathbf{\Sigma}_V = \begin{bmatrix} \sigma_{s,1}^{DDS} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{s,4}^{DDS} \end{bmatrix} \quad (32)$$

$\mu_{s,i}^{DDS}$  and  $\sigma_{s,i}^{DDS}$  are in dependency of the damage vector  $\delta$  (see Eq. 33).

$$\begin{aligned} \mu_{V,i}^{DDS} &= 1.0 + 0.1 \cdot \delta_i(t) \\ \sigma_{V,i}^{DDS} &= 0.7 - 0.01 \cdot \delta_i(t) \end{aligned} \quad (33)$$

The DIV threshold  $th_V$  is associated with the probability of false system damage indication  $PFI_S$  of 0.001.

### 5.3. Costs model

The costs considered in this study are the inspection costs  $C_I$ , the SHM costs  $C_{SHM}$ , the DDS installation and operating costs  $C_{DDS}$ , the repair costs  $C_R$ , the component fatigue failure costs  $C_{F,i}$ , and the system failure costs  $C_{FS}$ . The cost model used in this study is shown in Table 3 following Thöns et al. (2015). The SHM cost  $C_{SHM}$  is consist of investment, installation, and operating costs of 5 channel SHM system and are determined following Thöns et al. (2014). The discount rate  $r$  is assumed 0.06.

Table 3: Cost models used in the case study.

Type	Cost
$C_I$	0.001
$C_{SHM}^{Inv}$	$1.33 \cdot 10^{-4}/\text{channel}$
$C_{SHM}^{Inst}$	$1.33 \cdot 10^{-4}/\text{channel}$
$C_{SHM}^{Op}$	$2 \cdot 10^{-4}/\text{year}$
$C_{DDS}$	0.002
$C_R$	0.01
$C_{F,i}$	1
$C_{FS}$	100

### 5.4. Results and Discussion

The first SIM strategy is to perform only inspections at all hotspots to ensure the annual system failure probability lower than the threshold of  $3 \cdot 10^{-5}$ . The cumulative system failure probability  $P(F_S)$



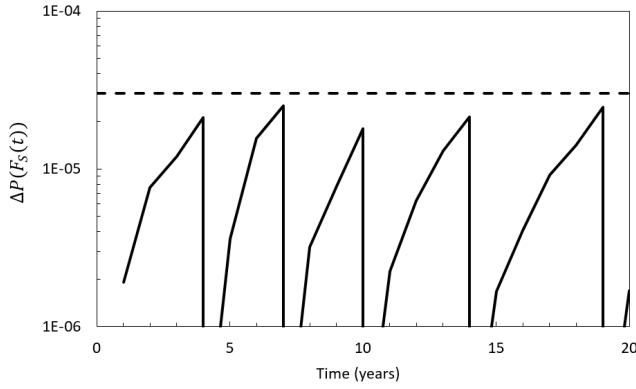


Figure 4: Annual system failure probability with only inspections.

over the service life is shown in Figure 4. The inspection times are at 4, 7, 10, 14, and 19 years. It is observed that the inspection interval is becoming longer with every inspection due to the updating of the system failure probability. The expected total costs  $C_{1,1}$  is 0.0637. In the prior scenario (without SIM), the expected total costs  $C_0$  is 0.0888.

The second SIM strategy is to install SHM system for a year before the first inspection is performed. The first and the second threshold ( $\eta_1, \eta_2$ ) of  $B_{\Delta S}$  are 0.79 and 1.07, respectively. The posterior system failure probability for each of monitoring outcomes is shown in Figure 5. If SHM indicates that the monitored component behaves better than designed ( $Z_1$ ), no further inspection in the future is needed. If SHM indicates that the monitored component behaves worse than designed ( $Z_3$ ), more inspections are required (6 in total) com-

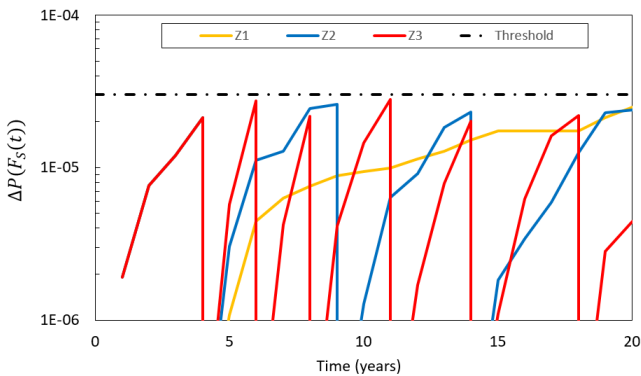


Figure 5: Annual system failure probability with SHM and inspections.

pared to inspection-only scenario. Reduction in inspection frequency is also observed if SHM indicates that the monitored component behaves as designed ( $Z_2$ ), where only 3 inspections are needed instead of 5 inspections. The expected total costs with this strategy  $C_{1,2}$  is 0.0535.

The third SIM strategy is to install DDS for one year before the first inspection. The posterior system failure probability for given DDS outcome can be seen in Figure 6. If no system damage is detected by DDS, the inspection interval is becoming longer but it does not change the number of required inspections (5 inspections). If DDS detects system damage, there is no observed change in inspection interval but the expected total becomes higher due to higher annual system failure probability. The expected total costs for inspections with DDS  $C_{1,3}$  is 0.0624.

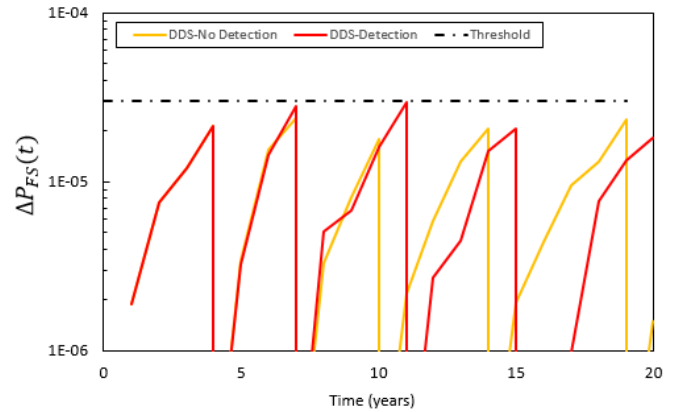


Figure 6: Annual system failure probability with DDS and inspections.

The Value of Information and Action (VoIA) of each strategy is shown in Figure 7 for three different system failure probability thresholds. From the figure, it can be seen that the VoIA has inverse relationship with system failure probability threshold. In a system with a low threshold, information from inspections, SHM, and DDS is essential to ensure the safety of the structural system and lower the annual system failure probability over the service life. In system with high threshold, the information from inspections, SHM, or DDS is only obtained in later part of the service life. Therefore, the value of information is lower due to discounting and lower remaining service life after the first inspection.



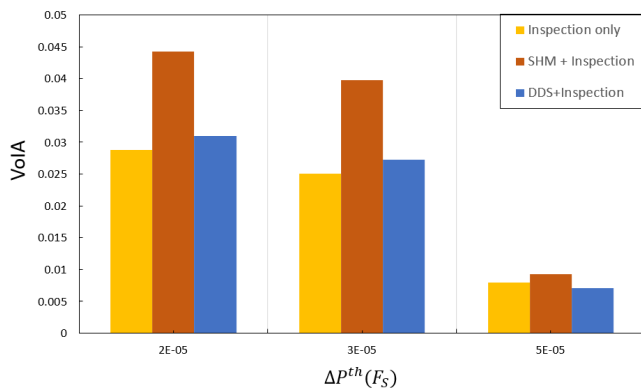


Figure 7: Value of Information and Action of each strategy for three different system failure probability thresholds.

From the computational costs point of view, the inspection-only strategy has the lowest computational costs due to the applied decision rule and the repair assumption. The third strategy (DDS+inspection) has the highest computational costs because of the complexity of the DIV distribution. The computational costs of the second strategy (SHM+inspection) is not as high as with DDS+inspection because the SHM thresholds are easier to calculate.

## 6. CONCLUSION

This paper addresses the quantification of the value of inspections, SHM, and DDS information by utilizing the Value of Information theory. A novel approach to predict SHM outcomes and update the system failure probability pre-posteriorly based on the stress range model uncertainty is introduced and combined with inspection planning to optimize SIM strategy. A deteriorating brittle Daniels system subjected to an extreme system load is used in the case study. Three different SIM strategies are investigated: inspections, inspections with SHM, and inspections with DDS. The results show that the information obtained from SHM and DDS could alter future inspection plans such as lowering/increasing the inspection frequency and reduces the expected total costs over the service life. It is also observed that the value of information and action (VoIA) of inspections, SHM, and DDS is decreasing with lower system failure probability threshold.

## 7. ACKNOWLEDGMENTS

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